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DEVELOPMENT OF THE *T*-YEAR MAXIMUM DISCHARGES ALONG THE DANUBE RIVER

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ABSTRACT

The territory of the Danube River Basin is one of the most flood-endangered regions in Europe. Therefore it is needed to have complete and comprehensive information about the river flood regime in order to be able to generalize this information on the basis of long-term observations from the whole Danube territory. Assessment of *T*-year maximum discharges belongs to the most important tasks for engineering hydrology. Several statistical methods are usually used for estimation of design discharges at gauging stations. Selection of an appropriate distribution function, method of parameter estimation, as well as selection of an analysed period often depends on the tradition in the country where they are used. To estimate the flood hazard along the streams such as Danube River it would be appropriate to use uniform methods for the *T*-year maximum discharges calculation. Therefore in this paper we present estimation of the maximum discharges with different return periods *T* (100, 200, 500, 1000 years) calculated according to log-Pearson Type III probability distribution. This theoretical type of the probability distribution is used to estimate the extremes in many natural processes and it is the most commonly used frequency distribution in hydrology. In this work the long-term annual maximum discharges from more than 20 stations along the Danube River were used to determine the *T*-year maximum discharges. At the end of this paper the development of the *T*-year maximum discharges along the Danube River is presented.

Keywords: annual maximum discharge, the Danube River, log-Pearson III distribution, *T*-year maximum discharge.

1 INTRODUCTION

One of the basic problems of the flood hydrology was (and still is) solution of the relationship between peak discharges of the flood waves and probability of their return period. Importance of extrapolation derived from these variables (so called frequency curve) is especially necessary for proposal of water management and flood control plans. Directive 2007/60/ EC of the European Parliament of 23 October 2007 concerning the assessment and management of flood risks requires member States to draw up flood hazard maps of floods with very long return periods *T* (500 to 1000 years). It is generally known, that the extrapolation of the data is very sensitive not only to the length of the data series, but also to the inclusion of the historic extremes to data series. The correct estimations of potential culminations of such floods require the inclusion of the longest data series of observations as well as the inclusion of historic pre-instrumental data to statistically analysed data series (Merz and Blöschl 2008a; Merz and Blöschl 2008b; Elleder 2010; Gaal et al. 2010; Elleder et al. 2013; Kjeldsen et al., 2014). Brazdil et al. (2006) studied historic hydrological materials in order to estimate floods threat in Europe. Estimation of the uncertainty at the design discharges was investigated for example by Szolgay et al. (2003); Merz et al. (2004) or Rogger et al. (2012). Except the mentioned factors the estimation of the *T*-year discharges is also influenced by used type of the theoretical probability distribution function. The choice of the type of the theoretical probability distribution function should relatively accurately represent uncertainty and variability of the hydrological problem. Application and choice of a particular probability distribution function, method of the parameter estimation as well as choice of the analysed period depend on the calculation method commonly used in a particular country. For large international basins such as the Danube River basin, it is necessary to synchronize the methodology and to prepare common procedures for determining flood hazard. Investigation of the history of extreme flood event frequency, severity and duration provides a greater understanding of the region's extreme event characteristics and the probability of recurrence at various levels of severity. This type of information is beneficial in the development of extreme response and mitigation strategies and preparedness plans.

In this study only the log-Pearson Type III distribution is used for the purpose of generalization of information about flood regime of the Danube River. The long-term maximum annual discharges from more than 20 water gauging stations along the Danube River are analysed and used for discharges with different return period estimation. Due to increasing length of time series of hydrological and meteorological data and their better availability, it is possible to work with the high number of data series. Some historical discharge extremes are included into the input data series. At the end of the paper estimated *T*-years maximum discharges with historical extremes and without historical extremes will be compared and presented in tables and graphs.

2 METHODS

Frequency analysis of the maximum discharges is a statistical approach for estimating the T-year discharges. The log-Pearson Type III distribution is one of the major distributions used in frequency analysis of the extremes. The log-Pearson Type III distribution is a statistical technique for fitting frequency distribution data to predict the design flood for a river at some site. Once the statistical information is calculated for the river site, the frequency distribution can be constructed. The probability of floods of various sizes can be extracted from the curve. The advantage of this particular technique is that extrapolation can be made of the values for events with return periods well beyond the observed flood events. This theoretical distribution belongs to the family of Pearson distributions, so called three parametric Gamma distributions, with logarithmic transformation of the data (Eq. 1):

$$y = lnx \;\rightarrow x = e^y \tag{1}$$

Parameters can be determined by several methods e.g.: LGMO – method of logarithmic moments, RLMO – method of real moments or MXM – method of mixed moments, more information you can see in Bobee (1975), or Rao (1980a, 1980b). The cumulative distribution function and probability distribution function according Hosking and Wallis (1997) are defined as:

If $\gamma \neq 0$ let $\alpha=4/\gamma^2$ and $\xi=\mu-2\sigma/\gamma$
If $\gamma>0$ then:

$$F(x) = G(\alpha, \frac{x-\xi}{\beta})/\Gamma(\alpha)\,, \tag{2}$$

$$f(x) = \frac{(x-\xi)^{\alpha-1}e^{-(x-\xi)/\beta}}{\beta^{\alpha}\Gamma(\alpha)}. \tag{3}$$

If $\gamma<0$ then

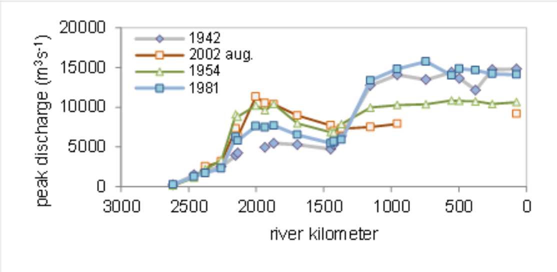
$$F(x) = 1 - G(\alpha, \frac{\xi-x}{\beta})/\Gamma(\alpha), \tag{4}$$

$$f(x) = \frac{(\xi-x)^{\alpha-1}e^{-(\xi-x)/\beta}}{\beta^{\alpha}\Gamma(\alpha)}, \tag{5}$$

where:
μ - location parameter;
σ - scale parameter;
γ - shape parameter;
Γ – Gamma function.

3 STUDY AREA AND DATA

The Danube River is the second greatest river in Europe. The basin covers an area of 817 000 km^2. The river originates from the Black Forest in Germany at the confluence of the Briga and the Breg streams. The Danube then discharges southeast for 2872 km, passing through four Central European capitals before emptying into the Black Sea via the Danube Delta in Romania and Ukraine. Based on geological composition the Danube River basin can be subdivided into three main parts, and the Danube delta: the Upper Danube Region, between the springs and the Devin Gate (Porta Hungarica), (131 338 km^2, 2051 m^{3}s^{-1}); the Central Danube Region between the Devin Gate and the Iron Gate (444 894 km^2, 5 585 m^{3}s^{-1} at Turnu Severin/Orsova gauge) and the Lower Danube Region, between the Iron Gate and the Danube's embouchure into the Black Sea (230 768 km^2, 6499 m^{3}s^{-1} at Ceatal Izmail gauge). There are 34 major tributaries of the Danube River. The Tisza River basin is the largest sub-basin in the basin (157 186 km^2). It is also the Danube's longest tributary (966 km). According to discharge volume, the Danube is the second largest river in Europe after the Volga River. The Sava River is the largest Danube tributary by discharge (average 1564 m^{3}s^{-1}) and the second largest by catchment area (95 419 km^2). The Inn is the third largest by discharge and the seventh longest Danube tributary. Given the size of the river basin, the large floods on the Danube usually do not occur simultaneously along the whole river. Some extreme floods along the Danube River are illustrated in Figure 1. The long-term data of the maximum annual discharges from more than 20 stations along the Danube River (Fig. 2) were used for the *T*-year maximum annual discharges assessment. The data were provided from the database of the project No. 9 "Flood regime of rivers in the Danube River Basin", within the IHP UNESCO Danube Cooperation. The list of selected gauging stations and basic hydrological and geographical characteristics are presented in Table 1.



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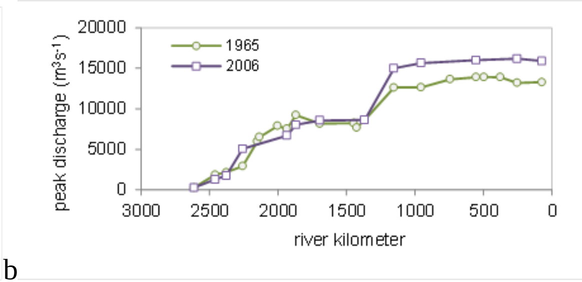

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Figure 1. Some extreme floods along the Danube River, a) floods in 1954, 2002 Aug. – significant at the Upper Danube River and floods in 1942, 1981 – significant at the Lower Danube River, b) floods in 1965, 2006 – significant along the entire length of the Danube River

Table 1. List of the gauging stations along the Danube River and Q_{amax} – long-term average of the maximum annual discharge

No.	River kilometer	Gauging station	Data series	Country	Area [km^2]	Zero gauge [a.s.l.]	Q_{amax} [m^{3}s^{-1}]
1	2613	Berg	1930–2007	GE	4047	489.48	204
2	2458.3	Ingolstadt	1940–2007	GE	20 001	359.97	1110
3	2376.1	Regensburg-Schwabelweis	1924–2007	GE	35 399	324.06	1532
4	2300	Pfelling	1926–2007	GE	37 757	307.73	1516
5	2256.9	Hofkirchen	1826–2013	GE	47 496	299.17	1896
6	2150	Achleiten	1901–2007	GE	76 653	287.27	4146
7	2135.2	Linz*	1821–2007	AT	79 490	247.06	3670
8	2002.7	Stein-Krems (Kienstock)*	1828–2006	AT	96 045	193.32	5372
9	1934.1	Wien-Nussdorf*	1828–2006	AT	101 731	157.0	5301
10	1868.8	Devin/Bratislava*	1876–2013	SK	131 338	132.86	5884
11	1694.6	Nagymaros	1893–2007	HU	183 534	99.37	5598
12	1446.8	Mohács	1930–2007	HU	209 064	79.19	5063
13	1425.5	Bezdan	1940–2006	SR	210 250	79.29	4974
14	1367.4	Bogojevo	1940–2006	SR	251 593	76.11	5675
15	1153.3	Pancevo	1940–2006	SR	525 009	65.98	10 147
16	1060	Veliko Gradiste	1931–2007	SR	570375	60.83	10 529
17	955	Orsova-Turnu Severin	1840–2006	RO	576 232	44.76	10 295
18	743.3	Lom	1942–1990	RO	588 860	22.89	10 632
19	554	Zimnicea	1931–2010	RO	658 400	16.06	11 087
20	495.6	Ruse	1940–1990	BG	669 900	11.99	11 116
21	375.5	Silistra	1941–1990	BG	689 700	6.5	11 009
22	252.3	Vadu Oii-Hirsova	1931–1990	RO	709 100	2.63	10 861
23	132	Reni	1921–2010	UKR	805 700	0.2	11 217
24	72	Ceatal Izmail*	1931–2010	RO	807 000		11 173

*T-year discharges were estimated both including extreme historical data as well as excluding historical data from 1501, 1787 and 1897

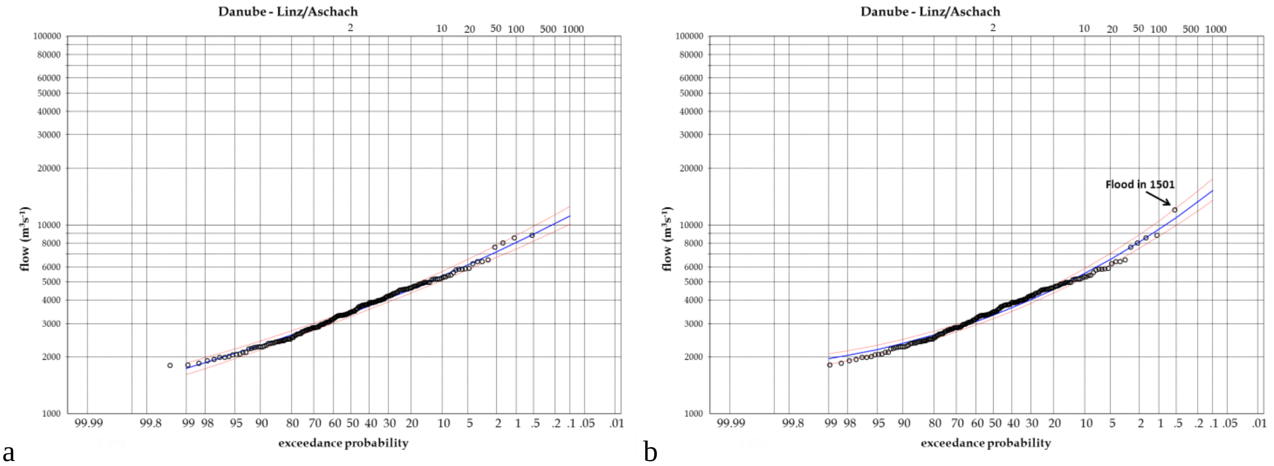
Figure 2. The Danube River basin and scheme of water gauging stations along the Danube River

4 RESULTS

This part of the paper presents the results of the maximum discharges estimation with different return period *T* (10, 50, 100, 200, 500 and 1000 years) according to the log-Pearson Type III distribution (LP III). Parameters of the LP III distribution were estimated by LGMO method. Empirical and theoretical frequency curves of the annual maxima discharges for selected 20 water gauging stations were estimated and evaluated. If some historical data in gauging station are known, the *T*-year maxima discharges were estimated again. Subsequently, courses and differences in the estimation of *T*-year maximum discharges in these stations were compared. Examples of such cases are presented on Figure 3 (gauging station Linz/Aschach) and Figure 4 (gauging station Ceatal Izmail).

Estimated maximum discharges for different return periods according to LP III distribution are listed in Table 2. The course of the maximum discharges with return period of 10, 100, 500 and 1000 years along the Danube River estimated according LP III distribution without and with including historical floods in 1501, 1787 and 1897 are presented on Figure 5 a-d.

Results show that the inclusion of the historical extremes to the estimation of the *T*-year discharges has no significant role for discharges with shorter return periods (5, 10, 20 years). But on the other hand, the estimation of the discharges with longer return period shows strong tail dependence of the LP III distribution on maximum values. The differences between estimated *T*-year maximum discharges, with and without inclusion of the historical extremes, reached about 3198 m^3s^{-1} at Q_{100} at Ceatal Izmail, and about 7300 m^3s^{-1} at Q_{1000} at Ceatal Izmail.



a b

Figure 3. Empirical and theoretical exceedance probability curve of the Danube maximum annual discharges at Linz/Aschach according to LP III distribution: a) Without historical maximum of the flood in 1501 (estimated max. 12 000 m^3s^{-1}) and b) With historical maximum of the flood in 1501 (estimated max. 12 000 m^3s^{-1})

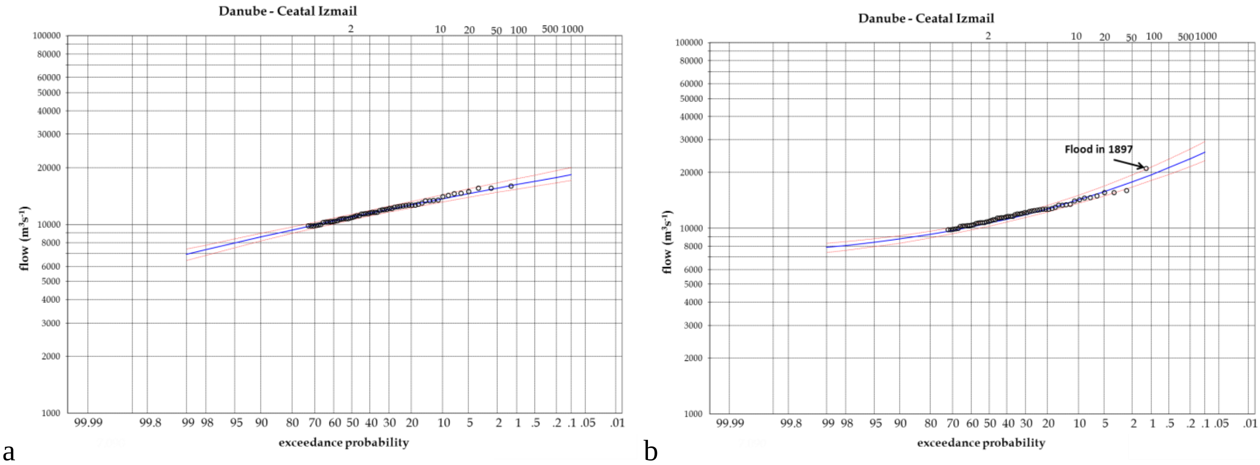


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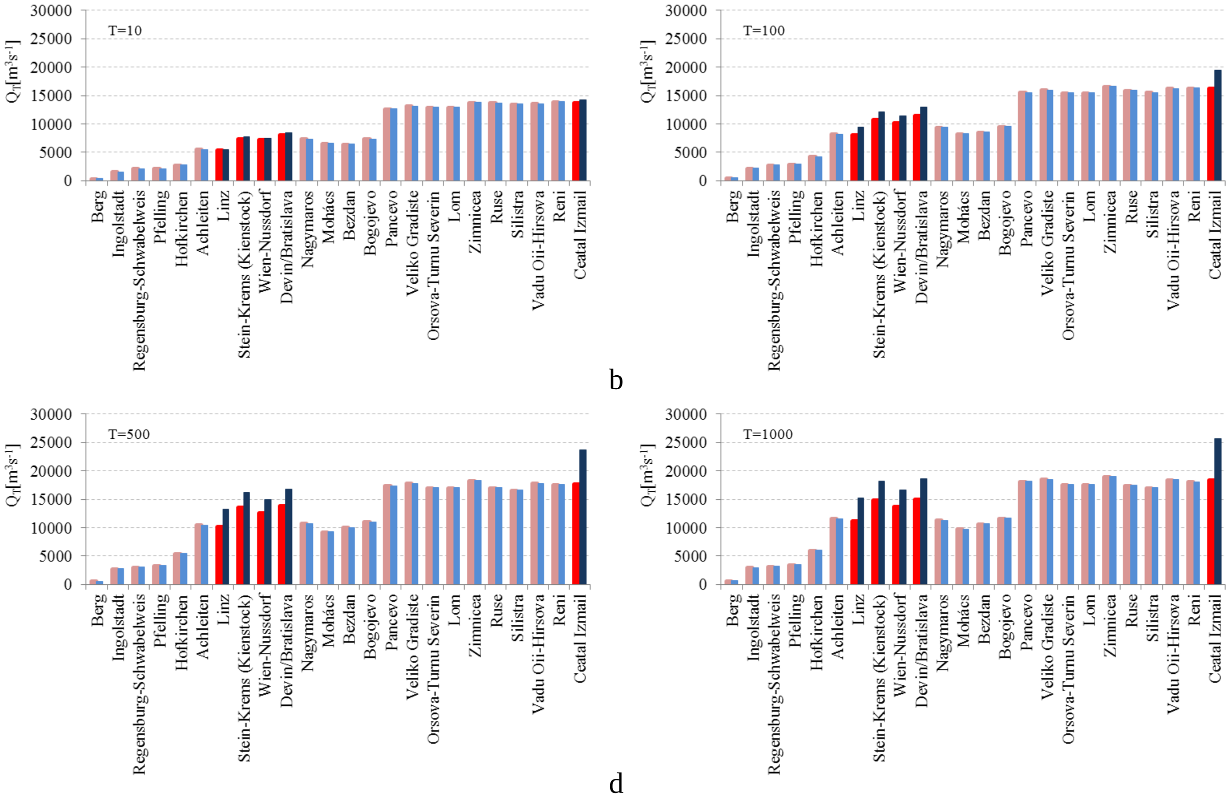
Figure 4. Empirical and theoretical exceedance probability curve of the Danube maximum annual discharges at Ceatal Izmail according to LP III distribution: a) without historical maximum of the flood in 1897 (estimated max. 20 940 m^3s^{-1}) and b) With historical maximum of the flood in 1897 (estimated max. 20 940 m^3s^{-1})



a b c d

Figure 5. Course of the maximum discharges with return period of 10, 100, 500 and 1000 years along the Danube River estimated according LP III distribution without and with including historical maxima estimated from floods in 1501, 1787 and 1897

5 CONCLUSION AND DISCUSION

The long-term annual data from more than 20 water gauging stations along the Danube River as well as some historical extremes were used to estimate the *T*-year maximum discharges. This paper was focused on estimation of the maximum discharges with different return period using the log-Pearson distribution Type III (LP III). Maximum discharges with different return period (5, 10, 20, 50, 100, 200, 500, 1000 years) were derived from theoretical curves according to LP III distribution. Some historical extreme data were including into the estimation of *T*-year discharges. Courses and differences between estimations, with and without historical extreme data, were compared. The results showed some differences in estimations especially for extreme values (discharges with longer return periods). Inclusion or non-inclusion of the historical extreme

Table 2. Estimated *T*-year maximal discharges at selected gauging stations along the Danube River (without historical maxima), LP III distribution

River kilometer	Return period [year]	5	10	50	100	200	500	1000
		Q_T – maximal discharge			[m^{3}s^{-1}]			
2613	Berg	272	325	434	478	521	577	618
2458.3	Ingolstadt	1329	1525	1986	2196	2415	2723	2970
2376.1	Regensburg-Schwabelweis	1901	2132	2564	2723	2870	3051	3180
2300	Pfelling	1888	2146	2664	2867	3062	3311	3494
2256.9	Hofkirchen	2342	2758	3762	4228	4721	5419	5986
2150	Achleiten	4815	5517	7303	8168	9106	10 470	11 608
2135.2	Linz*	**4557**	**5352**	**7215**	**8059**	**8939**	**10 166**	**11 149**
			5519	**8131**	**9472**	**10 973**	**13 243**	**15 206**
2002.7	Stein-Krems (Kienstock)*	**6423**	**7412**	**9759**	**10 834**	**11 962**	**13 548**	**14 828**
			7684	**10 685**	**12 162**	**13 776**	**16 150**	**18 150**
1934.1	Wien-Nussdorf*	**6318**	**7216**	**9309**	**10 253**	**11 235**	**12 602**	**13 696**
			7463	**10 159**	**11 469**	**12 890**	**14 961**	**16 693**
1868.8	Devin/Bratislava*	**7114**	**8138**	**10 443**	**11 451**	**12 482**	**13 892**	**14 999**
			8462	**11 521**	**12 980**	**14 545**	**16 802**	**18 667**
1694.6	Nagymaros	6621	7343	8817	9410	9990	10 744	11 308
1446.8	Mohács	5955	6558	7762	8236	8695	9284	9719
1425.5	Bezdan	5800	6459	7905	8525	9152	9999	10 657
1367.4	Bogojevo	6625	7344	8882	9526	10 169	11 025	11 681
1153.3	Pancevo	11 601	12 614	14 681	15 512	16 324	17 381	18 173
1060	Veliko Gradiste	12 093	13 132	15 190	15 996	16 773	17 770	18 506
955	Orsova-Turnu Severin	11 912	12 908	14 794	15 501	16 168	17 001	17 601
743.3	Lom	11 912	12 908	14 794	15 501	16 168	17 001	17 601
554	Zimnicea	12 733	13 786	15 828	16 611	17 359	18 306	18 999
495.6	Ruse	12 773	13 686	15 302	15 872	16 392	17 017	17 452
375.5	Silistra	12 601	13 467	14 992	15 526	16 012	16 595	17 000
252.3	Vadu Oii-Hirsova	12 519	13 545	15 492	16 226	16 920	17 788	18 415
132	Reni	12 960	13 943	15 705	16 334	16 910	17 609	18 098
72	Ceatal Izmail*	**12 694**	**13 693**	**15 569**	**16 270**	**16 928**	**17 748**	**18 338**
			14 239	**17 818**	**19 468**	**21 206**	**23 656**	**25 640**

*Without/with including estimated historical flood discharges in 1501. 1787 and 1897.

had no effect on estimation of the discharges with shorter return periods. Therefore we can conclude that this distribution is suitable for estimating the *T*–year maximum discharge with a longer return period.

The LP III distribution is used to estimate the extremes in many natural processes and is the most commonly used frequency distribution especially in hydrology. Pilon and Adamowski (1993) developed the Log Likelihood function of LPIII and estimated its parameters. Cheng et al. (2007) presented a frequency factor based method in hydrological frequency analysis for random generation of five distributions (normal. lognormal. extreme value type 1. Pearson Type III and log-Pearson Type III). Griffis and Stendinger (2007. 2009) used LP III in flood frequency analysis too. Log-Pearson Type III distribution is for example the distribution of choice for flood since 1976 in the USA (Koutsoyiannis 2008). Some authors (Vogel et al. 1993; Nazemi et al. 2011 or Stedinger and Griffis 2008) preferred the Generalized Extreme Value distribution (GEV). Comparison of several types of distributions (GEV. LPIII and Gumbel) for estimating T-year discharges presented Millington et al. (2011). Authors did not prefer any distribution as better and they suggested other researches in this problem. Phien and Jivajirajah (1984) dealt with the using of the log-Pearson III distribution to estimate maximum annual rainfall and discharges. They concluded that this distribution is more suitable for discharges with a longer return period (100 to 1000 years) but for the floods with shorter return period (1 to 50 years) in some cases there may be some uncertainty in estimations.

Natural climate fluctuations as well as expected climate changes bring a number of serious issues into the forecasting of the future river regime. In literature and media we encounter with information on more frequent catastrophic flooding and that drought are becoming of longer duration due to climate change. Monitoring and evaluation of extreme hydrological phenomena in the form of floods or droughts using various methods and models is very timely. Using one type of distribution also allows to estimate the value of the *T*-year maximum discharges in parts of the river without observations. Only on the basis of long-term average of maximum annual discharge and distribution parameters from the neighbouring gauging stations.

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